Part 1. Please create mathematical models using polynomial regression. Specifically:

|  |  |
| --- | --- |
| X (cm) | Y(cm) |
| 0 | 0 |
| 0.4 | 1 |
| 2 | 2.8 |
| 3.5 | 3.5 |
| 5 | 3.5 |
| 6 | 2.3 |
| 6.8 | 1.5 |
| 8 | 1 |
| 9 | 0.75 |
| 10 | 0.6 |
| 11 | 0.55 |
| 12 | 0.5 |
| 13 | 0.5 |
| 14 | 0.8 |
| 15 | 0.6 |
| 16 | 0.5 |

Create fitted polynomial models of orders 3,5,9,10 and 15. Also, calculate the regression coefficient, r, for each model to assess model fit to the data. What do you observe about the models as the polynomial model order increases? Explain whether any of these are satisfactory, i.e. match the artist’s drawing well. For each model that you determine is satisfactory, calculate the 2x area under the model, which is proportional to amount of material needed for 1 dessert.



correlation function









correlation function









correlation function













correlation function













correlation function







Analysis

As expected, the polynomial models fit the data better as the order increases as can be seen by the correlation coefficients and the visual fit. As the order increases from 9 to 10, the correlation coefficient increases, but oscillations occur that make the model unsatisfactory. This issue of oscillations is a significant limitation of regression models as the order is increased to achieve better correlation values. At an order of 15, the truncation error in the computations produces models with unacceptable error.

Hence, I would choose the best fit polynomial model as the 9th order due to both visual fit/lack of oscillations as well as high correlation factor. Note that there may be an artistic issue, since the model does not go through every data point. This is due to the tendency of statistical regression to ‘average’ data values in model fitting. This might be fixed by getting more data points, but would result in doing more computations to fit the model and potentially more oscillations.

Part 2. Please create mathematical function using a cubic spline model. Explain whether this model is satisfactory, i.e. matches the artist’s drawing well. Calculate the 2 x area under the model, which is proportional to amount of material needed for 1 dessert.























The spline coefficients are:



Note: a coefficients are the y data points

define spline equations for plotting

 

 

 

 

 

 

 

 

 

 

 

 

 

 

 

Analysis



As expected the cubic spline model goes through every data point smoothly. Artistically, the spline is an excellent fit to the data since it goes through all the data points. I note that there is little oscillation in this model. Note that oscillations can occur depending on data point/range choices.

Area calculations are below. These were done computationally, but you can simply integrate the cubic spline eqns analytically to obtain the same results.











The integral from z= xi to xi+1 for spline i is:



3. The chef plans to make 1000 of these desserts a year selling at $1000 per dessert. Raw materials cost $15 \* area per dessert (in cm2). Calculate how much profit she will make annually, comparing your choice of polynomial model and the cubic spline model.



Comparing areas and annual profits between the 9th order polynomial and cubic spline model:

9th order model has a bit smaller area (49.152 cm2) vs. the spline (49.494 cm2) and therefore the annual profit is slightly higher for the polynomial model ($262,700) vs. the polynomial model ($259,300).

However, given the small difference and the chef’s strong artistic preference, I would choose the cubic spline model as being more satisfactory.

Notes:

1. The difference in area appear to be primarily due to the model fit at the peaks/maximum values, where the spline has slightly larger areas. One way to interpret this is that the statistical nature of regression tends to flatten out maxima/minima.
2. Doing manual calculations are quite tedious, hence the value of computers/computational programs.

Using MathCad, this entire spline calculation is only 2 lines (see below).







